FIAN/TD/19-03
hep-th/0312297

Massive totally symmetric fields in AdS(d)

R.R. Metsaev*

Department of Theoretical Physics, P.N. Lebedev Physical Institute,
Leninsky prospect 53, Moscow 119991, Russia

Abstract

Free totally symmetric arbitrary spin massive bosonic and fermionic fields propagating in AdS(d) are investigated. Using the light cone formulation of relativistic dynamics we study bosonic and fermionic fields on an equal footing. Light-cone gauge actions for such fields are constructed. Interrelation between the lowest eigenvalue of the energy operator and standard mass parameter for arbitrary type of symmetry massive field is derived.

*E-mail: metsaev@lpi.ru

1 Introduction

A study of higher spin theories in AdS space-time has two main motivations (see e.g. [1, 2]): Firstly to overcome the well-known barrier of $N \leq 8$ in $d = 4$ supergravity models and, secondly, to investigate if there is a most symmetric phase of superstring theory that leads to the usual string theory as a result of a certain spontaneous breakdown of higher spin symmetries. Another motivation came from conjectured duality of a conformal $\mathcal{N} = 4$ SYM theory and a theory of type IIB superstring in $AdS_5 \times S^5$ background [3]. Recent discussion of this theme in the context of various limits in AdS superstring may be found in [4]. As is well known, quantization of GS superstring propagating in flat space is straightforward only in the light-cone gauge. It is the light-cone gauge that removes unphysical degrees of freedom explicitly and reduces the action to quadratical form in string coordinates. The light-cone gauge in string theory implies the corresponding light-cone formulation for target space fields. In the case of strings in AdS background this suggests that we should study a light-cone form dynamics of *target space fields* propagating in AdS space-time. It is expected that AdS massive fields form spectrum of states of AdS strings. Therefore understanding a light-cone description of AdS massive target space fields might be helpful in discussion of various aspects of AdS string dynamics. This is what we are doing in this paper.

Let us first formulate the main problem we solve in this letter. Fields propagating in AdS_d space are associated with positive-energy unitary lowest weight representations of $SO(d-1,2)$ group. A positive-energy lowest weight irreducible representation of the $SO(d-1,2)$ group denoted as $D(E_0, \mathbf{h})$, is defined by E_0, the lowest eigenvalue of the energy operator, and by $\mathbf{h} = (h_1, \ldots h_\nu)$, $\nu = [\frac{d-1}{2}]$, which is the highest weight of the unitary representation of the $SO(d-1)$ group. The highest weights h_i are integers and half-integers for bosonic and fermionic fields respectively. In this paper we investigate the fields whose E_0 and $\mathbf{h}$ are given by

$$E_0 > h_1 + d - 3\,, \qquad \mathbf{h} = (h_1, 0, \ldots, 0) \tag{1.1}$$

$$E_0 > h_1 + d - 3\,, \qquad \mathbf{h} = \begin{cases} (h_1, \frac{1}{2}, \ldots, \frac{1}{2}, \frac{1}{2}) & d - \text{ even} \\ (h_1, \frac{1}{2}, \ldots, \frac{1}{2}, \pm\frac{1}{2}) & d - \text{ odd} \end{cases} \tag{1.2}$$

The fields in (1.1) are massive bosonic fields while the ones in (1.2) are fermionic fields. The massive fields in (1.1),(1.2) with $h_1 \geq 1$, are referred to as totally symmetric fields[1]. In manifestly Lorentz covariant formulation the bosonic(fermionic) totally symmetric massive representation is described by a set of the tensor(tensor-spinor) fields whose $SO(d-1,1)$ space-time tensor indices have the structure of the respective Young tableauxes with one row. Covariant actions for the *bosonic* totally symmetric massive fields in AdS_d space were found in [10].[2]. *Fermionic* massive totally symmetry AdS_d fields with arbitrary E_0 and $\mathbf{h}$ have not been described at the field theoretical level so far [3]. In this paper we develop a light-cone gauge formulation for such fields at the action level.

[1] We note that the case $\mathbf{h} = (1, 0, \ldots, 0)$ corresponds to spin one massive field, the case $\mathbf{h} = (2, 0, \ldots, 0)$ is the massive spin two field.

[2] Massive self-dual spin fields in AdS_3 were investigated in [8]. Spin two massive fields were studied in [17]. Discussion of massive totally symmetric fields in $(A)dS_d$, $d \geq 4$, may be found in [9].

[3] Group theoretical description of various massive representation via oscillator method may be found, e.g., in [11]. Lorentz covariant equations of motion for AdS_5 self-dual massive fields with special values

Using a new version of the old light-cone gauge formalism in AdS space developed in [7], we describe both the bosonic and fermionic fields on an equal footing. Since, by analogy with flat space, we expect that a quantization of the Green-Schwarz AdS superstring with Ramond-Ramond charge will be straightforward only in the light-cone gauge [14] it seems that from the stringy perspective of AdS/CFT correspondence the light-cone approach is the fruitful direction to go.

2 Light-cone gauge action and its global symmetries

In this section we present new version of the old light cone formalism developed in [7]. Let $\phi(x)$ be a bosonic arbitrary spin field propagating in AdS_d space. If we collect spin degrees of freedom in a ket-vector $|\phi\rangle$ then a light-cone gauge action for ϕ can be cast into the following 'covariant form'[7][4]

$$S_{l.c.} = \frac{1}{2}\int d^dx \langle\phi^\dagger|(\Box - \frac{1}{z^2}A)|\phi\rangle\,, \qquad \Box = 2\partial^+\partial^- + \partial_i^2 + \partial_z^2\,. \tag{2.3}$$

An operator A does not depend on space-time coordinates and their derivatives. This operator acts only on spin indices collected in ket-vector $|\phi\rangle$. We call the operator A the AdS mass operator.

We turn now to discussion of global $so(d-1,2)$ symmetries of the light-cone gauge action. The choice of the light-cone gauge spoils the manifest global symmetries, and in order to demonstrate that these global invariances are still present one needs to find the Noether charges which generate them[5]. Noether charges (or generators) can be split into kinematical and dynamical generators. For $x^+ = 0$ the kinematical generators are quadratic in the physical field $|\phi\rangle$, while the dynamical generators receive corrections in interaction theory. In this paper we deal with free fields. At a quadratic level both kinematical and dynamical generators have the following standard representation in terms of the physical light cone field [7]

$$\hat{G} = \int dx^- d^{d-2}x \langle\partial^+\phi|G|\phi\rangle\,. \tag{2.4}$$

Representation for the kinematical generators in terms of differential operators G acting on the physical field $|\phi\rangle$ is given by

$$P^i = \partial^i\,, \qquad P^+ = \partial^+\,, \tag{2.5}$$

$$D = x^+P^- + x^-\partial^+ + x^I\partial^I + \frac{d-2}{2}\,, \tag{2.6}$$

$$J^{+-} = x^+P^- - x^-\partial^+\,, \tag{2.7}$$

---

of E_0 were discussed in [12]. Light cone actions for AdS_5 self-dual massive fields with arbitrary values of E_0 were found in [13].

[4] We use parametrization of AdS_d space in which $ds^2 = (-dx_0^2 + dx_i^2 + dx_{d-1}^2 + dz^2)/z^2$. Light-cone coordinates in $\pm$ directions are defined as $x^\pm = (x^{d-1} \pm x^0)/\sqrt{2}$ and x^+ is taken to be a light-cone time. We adopt the conventions: $\partial^i = \partial_i \equiv \partial/\partial x^i$, $\partial_z \equiv \partial/\partial z$, $\partial^\pm = \partial_\mp \equiv \partial/\partial x^\mp$, $z \equiv x^{d-2}$ and use indices $i,j = 1,\ldots,d-3$; $I,J = 1,\ldots,d-2$. Vectors of $so(d-2)$ algebra are decomposed as $X^I = (X^i, X^z)$.

[5] These charges play a crucial role in formulating interaction vertices in field theory. Application of Noether charges in formulating superstring field theories may be found in [15]

$$J^{+i} = x^+\partial^i - x^i\partial^+\,, \tag{2.8}$$

$$J^{ij} = x^i\partial^j - x^j\partial^i + M^{ij}\,, \tag{2.9}$$

$$K^+ = -\frac{1}{2}(2x^+x^- + x^Jx^J)\partial^+ + x^+D\,, \tag{2.10}$$

$$K^i = -\frac{1}{2}(2x^+x^- + x^Jx^J)\partial^i + x^iD + M^{iJ}x^J + M^{i-}x^+\,, \tag{2.11}$$

while a representation for the dynamical generators takes the form

$$P^- = -\frac{\partial_J^2}{2\partial^+} + \frac{1}{2z^2\partial^+}A\,, \tag{2.12}$$

$$J^{-i} = x^-\partial^i - x^iP^- + M^{-i}\,, \tag{2.13}$$

$$K^- = -\frac{1}{2}(2x^+x^- + x_I^2)P^- + x^-D + \frac{1}{\partial^+}x^I\partial^JM^{IJ} - \frac{x^i}{2z\partial^+}[M^{zi},A] + \frac{1}{\partial^+}B\,, \tag{2.14}$$

where $M^{-i} = -M^{i-}$ and

$$M^{-i} \equiv M^{iJ}\frac{\partial^J}{\partial^+} - \frac{1}{2z\partial^+}[M^{zi},A]\,. \tag{2.15}$$

Operators A, B, M^{IJ} are acting only on spin degrees of freedom of wave function $|\phi\rangle$. $M^{IJ} = M^{ij}, M^{zi}$ are spin operators of the $so(d-2)$ algebra

$$[M^{IJ},M^{KL}] = \delta^{JK}M^{IL} + 3 \text{ terms}\,, \tag{2.16}$$

while the operators A and B admit the following representation

$$A = \frac{1}{2}M^{IJ}M^{IJ} + \frac{d(d-2)}{4} - \langle Q_{AdS}\rangle + 2B^z + M^{zi}M^{zi}\,, \tag{2.17}$$

$$B = B^z + M^{zi}M^{zi}\,. \tag{2.18}$$

$\langle Q_{AdS}\rangle$ is eigenvalue of the second order Casimir operator of the $so(d-1,2)$ algebra for the representation labelled by $D(E_0,\mathbf{h})$:

$$-\langle Q_{AdS}\rangle = E_0(E_0+1-d) + \sum_{\sigma=1}^{\nu} h_\sigma(h_\sigma - 2\sigma + d - 1)\,, \tag{2.19}$$

while B^z is z-component of $so(d-2)$ algebra vector B^I which satisfies the defining equation[6]

$$[B^I,B^J] + (M^3)^{[I|J]} + (\langle Q_{AdS}\rangle - \frac{1}{2}M^2 - \frac{N(N-1)+2}{2})M^{IJ} \approx 0\,. \tag{2.20}$$

Here we use sign $\approx$ to write instead of equations $X|\phi\rangle = 0$ simplified equations $X \approx 0$. As was noted the operator B^I transforms in vector representation of the $so(d-2)$ algebra

$$[B^I,M^{JK}] = \delta^{IJ}B^K - \delta^{IK}B^J\,. \tag{2.21}$$

[6]We use the notation $(M^3)^{[I|J]} \equiv \frac{1}{2}M^{IK}M^{KL}M^{LJ} - (I \leftrightarrow J)$, $M^2 \equiv M^{IJ}M^{IJ}$. Throughout this paper we use a convention $N \equiv d-2$.

Making use of the formulas above-given one can check that the light-cone gauge action (2.3) is invariant with respect to the global symmetries generated by $so(d-1,2)$ algebra taken to be in the form $\delta_{\hat{G}}|\phi\rangle = G|\phi\rangle$. To summarize a procedure of finding light cone description consists of the following steps:

i) choose form of realization of spin degrees of freedom of the field $|\phi\rangle$

ii) fix appropriate representation for the spin operator M^{IJ};

iii) using formula (2.19) evaluate an eigenvalue of the Casimir operator;

iv) find solution to the defining equations for the operator B^I (2.20).

Now following this procedure we discuss bosonic and fermionic fields in turn.

3 Bosonic fields

To discuss field theoretical description of massive AdS field we could use $so(d-1)$ totally symmetric traceless tensor field $\Phi^{\hat{I}_1\ldots\hat{I}_s}$, $\hat{I}=1',1,\ldots,d-2$. Instead of this we prefer to decompose such field into traceless totally symmetric tensors of $so(d-2)$ algebra $\phi^{I_1\ldots I_{s'}}$, $I=1,\ldots,d-2$; $s'=0,1,\ldots,s$:

$$\Phi^{\hat{I}_1\ldots\hat{I}_s} = \sum_{s'=0}^{s} \oplus\, \phi^{I_1\ldots I_{s'}}\,. \tag{3.22}$$

As usual to avoid cumbersome tensor expressions we introduce creation and annihilation oscillators α^I and $\bar{\alpha}^I$

$$[\bar{\alpha}^I,\,\alpha^J]=\delta^{IJ}\,,\qquad \bar{\alpha}^I|0\rangle = 0\,, \tag{3.23}$$

and make use of ket-vectors $|\phi_{s'}\rangle$ defined by

$$|\phi_{s'}\rangle \equiv \alpha^{I_1}\ldots\alpha^{I_{s'}}\phi^{I_1\ldots I_{s'}}(x)|0\rangle\,. \tag{3.24}$$

The $|\phi_{s'}\rangle$ satisfies the following algebraic constraints

$$(\alpha\bar{\alpha}-s')|\phi_{s'}\rangle = 0\,,\qquad \alpha\bar{\alpha}\equiv\alpha^I\bar{\alpha}^I \tag{3.25}$$

$$\bar{\alpha}^I\bar{\alpha}^I|\phi_{s'}\rangle = 0\,,\qquad \text{tracelessness}\,. \tag{3.26}$$

Eq.(3.25) tells us that $|\phi_{s'}\rangle$ is a polynomial of degree s' in oscillator α^I. Tracelessness of the tensor fields $\phi^{I_1\ldots I_{s'}}$ is reflected in (3.26). The spin operator M^{IJ} of the $so(d-2)$ algebra for the above defined fields $|\phi_{s'}\rangle$ takes then the form

$$M^{IJ} = \alpha^I\bar{\alpha}^J - \alpha^J\bar{\alpha}^I\,. \tag{3.27}$$

We are going to connect our spin s field with the unitary representation labelled by $D(E_0,\mathbf{h})$ (1.1) whose h_1 is identified with spin value s:[7]

$$h_1 = s\,. \tag{3.28}$$

Eigenvalue of the second order Casimir operator for totally symmetric representation under consideration is given then by (see (1.1),(2.19),(3.28))

$$-\langle Q_{AdS}\rangle = E_0(E_0+1-d) + s(s+d-3)\,. \tag{3.29}$$

[7]This identification can be proved rigourously by exploiting the procedure of Sec.6 in Ref.[13].

An action of the operator B^I on the physical fields $|\phi_{s'}\rangle$ is found to be

$$B^I|\phi_{s'}\rangle = a_{s'}A^I_{s'-1}|\phi_{s'-1}\rangle + b_{s'}\bar{\alpha}^I|\phi_{s'+1}\rangle\,, \tag{3.30}$$

where the coefficients $a_{s'}$, $b_{s'}$ depend on E_0, s and s'

$$a_{s'} = b(E_0, s, s'-1)\,, \qquad b_{s'} = b(E_0, s, s')\,, \tag{3.31}$$

and a function $b(E_0, s, s')$ is defined to be

$$b(E_0, s, s') \equiv \left(\frac{(s-s')(s+s'+N-1)(E_0-s'-N)(E_0+s'-1)}{2s'+N}\right)^{1/2}\,. \tag{3.32}$$

Operator $A^I_{s'}$ which appears in definition of the operator B^I (3.30) is given by

$$A^I_{s'} \equiv \alpha^I - \frac{\alpha^2\bar{\alpha}^I}{2s'+N-2}\,, \qquad \alpha^2 \equiv \alpha^I\alpha^I\,. \tag{3.33}$$

Let us outline procedure of derivation above-mentioned results. Most difficult problem is to find solution to the defining equation (2.20). Fortunately, for the case of totally symmetric fields this equation simplifies due to the following relation for M^{IJ}:

$$(M^3)^{[I|J]} = (-\frac{1}{2}M^2 + \frac{(N-2)(N-3)}{2})M^{IJ}\,. \tag{3.34}$$

This formula can be checked directly by using representation for M^{IJ} given in (3.27). Plugging (3.34) in (2.20) we get the following simplified form of defining equation

$$[B^I, B^J] + (\langle Q_{AdS}\rangle - M^2 - 2N + 2)M^{IJ} \approx 0\,. \tag{3.35}$$

Applying this equation to $|\phi_{s'}\rangle$ and using (3.30) we get a relationship for the coefficients $a_{s'}$ and $b_{s'}$

$$a_{s'+1}b_{s'} = \frac{(s-s')(s+s'+N-1)(E_0-s'-N)(E_0+s'-1)}{2s'+N}\,. \tag{3.36}$$

Now we exploit a requirement the operator B^I be hermitian[8]

$$\langle\psi||B^I\phi\rangle = \langle B^I\psi||\phi\rangle\,, \tag{3.37}$$

with respect to scalar product defined by

$$\langle\psi||\phi\rangle \equiv \sum_{s'=0}^{s}\langle\psi_{s'}||\phi_{s'}\rangle\,. \tag{3.38}$$

This requirement gives the relation $a_{s'} = b^*_{s'-1}$. Making use of this relation in (3.36) we arrive at the final solution given in (3.31). Some helpful formulas to evaluate commutator $[B^I, B^J]$ are given by

$$A^I_{s'}A^J_{s'-1} - (I \leftrightarrow J) = 0\,, \tag{3.39}$$

$$A^I_{s'}\bar{\alpha}^J - (I \leftrightarrow J) = M^{IJ}\,, \tag{3.40}$$

$$\bar{\alpha}^I A^J_{s'} - (I \leftrightarrow J) = -\frac{2s'+N}{2s'+N-2}M^{IJ}\,. \tag{3.41}$$

[8]We use anti-hermitian representation for generators of $so(d-1,2)$ algebra (2.4). This implies that the spin operator M^{IJ} should be anti-hermitian, while the operators A and B^I should be hermitian.

4 Fermionic fields

Light cone action for fermionic fields takes the following form

$$S_{l.c.}^{ferm} = \int d^d x \langle\psi^\dagger| \frac{\mathrm{i}}{2\partial^+}(\Box - \frac{1}{z^2}A)|\psi\rangle\,. \qquad (4.42)$$

This action is invariant with respect to transformations $\delta|\psi\rangle = G^{ferm}|\psi\rangle$, where differential operators G^{ferm} are obtainable from the ones for bosonic fields ((2.5)-(2.14)) by making there the following substitution $x^- \rightarrow x^- + \frac{1}{2\partial^+}$. In addition to this in expressions for generators in ((2.5)-(2.14)) we should use the spin operator M^{IJ} suitable for fermionic fields. Defining equation (2.20) for the operator B^I does not change. Before to discuss concrete form of the spin operators M^{IJ} and B^I we should fix a field theoretical realization of spin degrees of freedom collected in $|\psi\rangle$.

To discuss field theoretical description of massive AdS field we could used totally symmetric traceless and γ-transversal tensor-spinor field $\Psi^{\hat{I}_1\ldots\hat{I}_s\alpha}$, $\hat{I} = 1', 1, \ldots, d-2$ which corresponds to irreducible spin $s+\frac{1}{2}$ representation of $so(d-1)$ algebra. Instead of this we prefer to decompose such field into traceless totally symmetric tensor-spinor fields of $so(d-2)$ algebra $\psi^{I_1\ldots I_{s'}\alpha}$, $I = 1,\ldots,d-2$; $s' = 0,1,\ldots,s$:[9]

$$\Psi^{\hat{I}_1\ldots\hat{I}_s\alpha} = \sum_{s'=0}^{s} \oplus\, \psi^{I_1\ldots I_{s'}\alpha}\,. \qquad (4.43)$$

As before to avoid cumbersome expressions we exploit creation and annihilation oscillators α^I and $\bar{\alpha}^I$ (3.23) and make use of ket-vectors $|\psi_{s'}\rangle$ defined by

$$|\psi_{s'}\rangle \equiv \alpha^{I_1}\ldots\alpha^{I_{s'}}\psi^{I_1\ldots I_{s'}\alpha}(x)|0\rangle\,. \qquad (4.44)$$

Here and below spinor indices are implicit. The $|\psi_{s'}\rangle$ satisfies the following algebraic constraints

$$(\alpha\bar{\alpha} - s')|\psi_{s'}\rangle = 0\,, \qquad (4.45)$$

$$\gamma^I\bar{\alpha}^I|\psi_{s'}\rangle = 0\,, \qquad \gamma - \text{transversality} \qquad (4.46)$$

$$\bar{\alpha}^I\bar{\alpha}^I|\psi_{s'}\rangle = 0\,, \qquad \text{tracelessness}\,. \qquad (4.47)$$

Eq. (4.45) tells us that $|\psi_{s'}\rangle$ is a polynomial of degree s' in oscillator α^I. Tracelessness of the tensor-spinor field $\psi^{I_1\ldots I_{s'}\alpha}$ is reflected in (4.47). Realization of spin operator M^{IJ} on the space of the ket-vectors $|\psi_{s'}\rangle$ is given by

$$M^{IJ} = M_b^{IJ} + \frac{1}{2}\gamma^{IJ}\,, \qquad M_b^{IJ} \equiv \alpha^I\bar{\alpha}^J - \alpha^J\bar{\alpha}^I\,, \qquad \gamma^{IJ} \equiv \frac{1}{2}(\gamma^I\gamma^J - \gamma^J\gamma^I)\,. \qquad (4.48)$$

We are going to connect our massive tensor-spinor field with unitary representation labelled by $D(E_0,\mathbf{h})$ (1.1) whose h_1 is related with integer s:

$$h_1 = s + \frac{1}{2}\,. \qquad (4.49)$$

[9]Details of such a decomposition may be found in Appendix B. The $\psi^{I_1\ldots I_{s'}\alpha}$ are obtainable from $2^{[d/2]}$ Dirac tensor-spinor fields of $so(d-1,1)$ algebra: $\psi = \frac{1}{2}\gamma^-\gamma^+\psi_{Dirac}$. We use $2^{[d/2]}\times 2^{[d/2]}$ -Dirac γ-matrices: $\{\gamma^a,\gamma^b\} = 2\eta^{ab}$, $\eta^{ab} = (-1,+1,\ldots,+1)$, $a,b = 0,1,\ldots,d-1$. In light cone frame we use a decomposition $\gamma^a = \gamma^+,\gamma^-,\gamma^I$, where $\gamma^\pm \equiv (\gamma^{d-1}\pm\gamma^0)/\sqrt{2}$. $I = 1,\ldots,d-2$.

Eigenvalue of the second order Casimir operator for totally symmetric fermionic representation is given then by (see (1.2),(2.19),(4.49))

$$-\langle Q_{AdS}\rangle = E_0(E_0-1-N)+s(s+N)+\frac{N(N+1)}{8} \qquad (4.50)$$

An action of the operator B^I on the physical fermionic fields $|\psi_{s'}\rangle$ is found to be

$$B^I|\psi_{s'}\rangle = q_{s'}\mathcal{Y}^I_{s'}|\psi_{s'}\rangle + a_{s'}\mathcal{A}^I_{s'-1}|\psi_{s'-1}\rangle + b_{s'}\bar{\alpha}^I|\psi_{s'+1}\rangle\,. \qquad (4.51)$$

As before the coefficients $q_{s'}$, $a_{s'}$, $b_{s'}$ turn out to be functions of E_0, s, s' and are given by

$$q_{s'} = \frac{2s+N}{2(2s'+N)}(E_0-\frac{N+1}{2})\,, \qquad (4.52)$$

$$a_{s'} = f(E_0,s,s'-1)\,, \qquad (4.53)$$

$$b_{s'} = f(E_0,s,s')\,, \qquad (4.54)$$

where we use the notation

$$f(E_0,s,s') \equiv \Bigl(\frac{(s-s')(s+s'+N)(E_0-s'-N-\frac{1}{2})(E_0+s'-\frac{1}{2})}{2s'+N}\Bigr)^{1/2}\,. \qquad (4.55)$$

Operators $\mathcal{A}^I_{s'}$ and $\mathcal{Y}^I_{s'}$ which enter definition of basic operator B^I (4.51) are given by

$$\mathcal{A}^I_{s'} \equiv \alpha^I - \frac{\alpha^2\bar{\alpha}^I+(\gamma\alpha)\gamma^I}{2s'+N}\,, \qquad \mathcal{Y}^I_{s'} \equiv \gamma^I - \frac{2(\gamma\alpha)\bar{\alpha}^I}{2s'+N-2}\,, \qquad \gamma\alpha \equiv \gamma^I\alpha^I\,. \qquad (4.56)$$

Now let us outline procedure derivation of these results. We start with general representation for B^I given in (4.51) and the problem is to find coefficients $q_{s'}$, $a_{s'}$, $b_{s'}$ which satisfy the defining equation (2.20). To this end we evaluate expression for $(M^3)^{[I|J]}$

$$\begin{aligned}(M^3)^{[I|J]} &\approx M_b^{IJ}\Bigl(s'(s'+N-\frac{1}{2})+\frac{2N(N-3)+5}{4}\Bigr)\\ &+ \gamma^{IJ}\Bigl(\frac{1}{2}s'+\frac{N(N-3)+3}{8}\Bigr)-\frac{2s'+N-1}{4}(\gamma\alpha)\bar{S}^{IJ}\,, \qquad (4.57)\\ \bar{S}^{IJ} &\equiv \gamma^I\bar{\alpha}^J-\gamma^J\bar{\alpha}^I\,,\end{aligned}$$

where a sign $\approx$ indicates that (4.57) is valid by module of terms which are equal to zero by applying to $|\psi_{s'}\rangle$ i.e. to derive (4.57) we use constraints (4.45)-(4.47). After this using Eqs.(2.20),(4.50),(4.57) we find solution for $q_{s'}$ and product $a_{s'+1}b_{s'}$. Exploiting then the hermicity condition for the operator B^I (3.37),(3.38) we arrive at the solution for $q_{s'}$, $a_{s'}$, $b_{s'}$ given above. Some helpful formulas to evaluate commutator $[B^I,B^J]$ are given by

$$\mathcal{A}^I_{s'}\mathcal{A}^J_{s'-1}-(I\leftrightarrow J)=0\,, \qquad (4.58)$$

$$[\mathcal{Y}^I_{s'},\mathcal{Y}^J_{s'}] = -\frac{4}{2s'+N-2}M_b^{IJ}+2\gamma^{IJ}+4\frac{2s'+N-1}{(2s'+N-2)^2}(\gamma\alpha)\bar{S}^{IJ}\,, \qquad (4.59)$$

$$\mathcal{A}^I_{s'}\bar{\alpha}^J-(I\leftrightarrow J) = M_b^{IJ}-\frac{(\gamma\alpha)\bar{S}^{IJ}}{2s'+N}\,, \qquad (4.60)$$

$$\bar{\alpha}^I\mathcal{A}^J_{s'}-(I\leftrightarrow J) = -\frac{2s'+N+2}{2s'+N}M_b^{IJ}-\frac{2\gamma^{IJ}}{2s'+N}+\frac{(\gamma\alpha)\bar{S}^{IJ}}{2s'+N}\,. \qquad (4.61)$$

5 Interrelation between lowest energy value E_0 and mass parameter m

In previous sections we have expressed our results in terms of lowest eigenvalue of energy operator E_0. Because sometimes formulation in terms of the standard mass parameter m is preferable we would like to derive interrelation between E_0 and m. Before to going into details let of first present our results.

Given massive fields with massive parameter m corresponding to unitary representation labelled by $D(E_0,\mathbf{h})$ we find the following relationship between E_0 and m (for even AdS space-time dimension d; $\nu=(d-2)/2$)

$$E_0=\frac{d-1}{2}+\sqrt{m^2+\Big(h_k-k+\frac{d-3}{2}\Big)^2}\,, \qquad \text{for bosonic fields;} \tag{5.62}$$

$$E_0=m+h_k-k-2+d\,, \qquad \text{for fermionic fields}\,, \tag{5.63}$$

where a number k is defined from the relation

$$h_1=\ldots=h_k>h_{k+1}\geq h_{k+2}\geq\ldots\geq h_\nu\geq 0. \tag{5.64}$$

We remind that for bosonic fields the labels h_σ are integers while for fermionic fields the h_σ are half-integers. We note that relations (5.62),(5.63) are valid also for those massive fields in odd dimensional AdS_d whose $h_{(d-1)/2}=0$.

Now let us outline procedure of derivation these results. Let $\phi_{m=0}^{\mu_1\cdots}$ be massless field in AdS_d. As was demonstrated in [5],[6] the massless fields associated with unitary representation labelled by $D(E_0^{m=0},\mathbf{h})$ should satisfy the equation of motion

$$(\mathcal{D}^2-E_0^{m=0}(E_0^{m=0}+1-d)+\sum_{\sigma=1}^{\nu}h_\sigma)\phi_{m=0}^{\mu_1\cdots}=0\,, \tag{5.65}$$

where $\mathcal{D}^2$ is a covariant D'Alembertian operator in AdS_d and $E_0^{m=0}$ is given by

$$E_0^{m=0}=h_k-k-2+d\,. \tag{5.66}$$

Eq. (5.65) reflects the well-known fact that equations of motion for AdS massless field involve mass-like term which is expressible in terms of $E_0^{m=0}$. As is well known to discuss gauge invariant description of massive fields one introduces the set of fields including some fields $\phi_m^{\mu_1\cdots}$ which we shall refer to as leading field plus Goldstone fields (sometimes referred to as Stueckelberg fields). By definition, the structure of Lorentz indices of the leading massive field $\phi_m^{\mu_1\cdots}$ is the same as the one for massless field $\phi_{m=0}^{\mu_1\cdots}$. If we impose on the leading field $\phi_m^{\mu_1\cdots}$ and the Goldstone fields an appropriate covariant Lorentz gauge and tracelessnes conditions then for the leading field one gets the equation

$$(\mathcal{D}^2-m^2-E_0^{m=0}(E_0^{m=0}+1-d)+\sum_{\sigma=1}^{\nu}h_\sigma)\phi_m^{\mu_1\cdots}=0\,. \tag{5.67}$$

One other hand an analysis of Ref.[5],[6] implies that the leading field satisfies the following equations of motion

$$(\mathcal{D}^2-E_0(E_0+1-d)+\sum_{\sigma=1}^{\nu}h_\sigma)\phi_m^{\mu_1\cdots}=0\,. \tag{5.68}$$

Comparison of Eqs. (5.67) and (5.68) gives a relationship between E_0 and m

$$m^2 = E_0(E_0 + 1 - d) - E_0^{m=0}(E_0^{m=0} + 1 - d)\,. \qquad (5.69)$$

Solution to this equation corresponding to positive values of E_0 is given in (5.62).

The same arguments can be applied to the fermionic fields. In this case equation for massless field in AdS_d is given by (see the second Refs. in [5])

$$(\gamma^a e_a^\mu D_{\mu L} + E_0^{m=0} + \frac{1-d}{2})\psi_{m=0}^{\alpha\,\mu_1\ldots} = 0\,, \qquad (5.70)$$

where $D_{\mu L}$ is a covariant derivative with respect to local Lorentz rotation, e_a^μ is an inverse of the vielbein e_μ^a and $E_0^{m=0}$ is given in (5.66). Because on the one hand the leading massive tensor-spinor field $\psi_m^{\alpha\,\mu_1\cdots}$ taken to be in appropriate covariant Lorentz gauge satisfies an equation

$$(\gamma^a e_a^\mu D_{\mu L} + m + E_0^{m=0} + \frac{1-d}{2})\psi_m^{\alpha\,\mu_1\cdots} = 0\,, \qquad (5.71)$$

and on the other hand this equation should be representable as (see [5])

$$(\gamma^a e_a^\mu D_{\mu L} + E_0 + \frac{1-d}{2})\psi_m^{\alpha\,\mu_1\cdots} = 0\,, \qquad (5.72)$$

we find the relation for m

$$m = E_0 - E_0^{m=0}\,, \qquad (5.73)$$

which together with (5.66) leads to (5.63).

In the AdS/CFT correspondence the E_0 is connected with dimension of conformal operator as $E_0 = \Delta$. The Δ for bosonic massive totally antisymmetric and bosonic massive symmetric spin two fields were evaluated in Refs.[16],[17]. Our results coincides with the ones obtained in these references. For instance for the case of bosonic massive totally symmetric fields we have $h_1 = s$, $k = 1$ and this leads to

$$E_0 = \Delta = \frac{d-1}{2} + \sqrt{m^2 + \Bigl(s + \frac{d-5}{2}\Bigr)^2}\,. \qquad (5.74)$$

For the case of $s = 2$ this is result of Ref.[17].

For fermionic massive totally symmetric fields we have $h_1 = s + \frac{1}{2}$, $k = 1$ and formula (5.63) leads to

$$E_0 = \Delta = m + s + d - \frac{5}{2}\,. \qquad (5.75)$$

For particular value of $s = 1$ (Rarita-Schwinger field) appropriate Δ was evaluated in Refs.[18],[19]. Note that our result taken to be for $s = 1$ differs from the one obtained in these references. The reason for this is that in this paper we use normalization of mass parameter such that the point $m = 0$ corresponds to massless fields (see Eqs. (5.70),(5.71)). In Refs.[18],[19] another normalization was used.

Because while derivation of our results for E_0 we used arguments based on gauge invariant formulation for massive fields our relations (5.62),(5.63) are not applicable to the scalar field and spin one-half field. Conformal dimension for these fields are well known (see [20],[21]).

Conclusions. The results presented here should have a number of interesting applications and generalizations, some of which are: i) In this paper we develop light cone formulation for massive totally symmetric fields. It would be interesting to extend such formulation to the study of massive mixed symmetry fields (see Refs.[22],[23],[24]) and then to apply such formulation to the study of AdS/CFT correspondence along the line of Ref.[25],[26]. ii) As is well known massive and massless fields can be connected via procedure of dimensional reduction. Procedure of dimensional reduction $AdS_d \rightarrow AdS_{d-1}$ was developed in [8] (for discussion of alternative reductions see [27],[28]). It would be interesting to find mass spectra of AdS massive modes upon dimensional reduction of massless AdS fields.

Acknowledgments. This work was supported by the INTAS project 03-51-6346, by the RFBR Grant No.02-02-17067, and RFBR Grant for Leading Scientific Schools, Grant No. 1578-2003-2.

Appendix A Interrelation between new and old light cone formulations

In this appendix we explain interrelation of basic defining equation for the operator B^I and representation for the operators A and B (2.17),(2.18) with the old defining equations given in Ref.[7]. Defining equations for the operators A and B take the form[7]

$$2\{M^{zi}, A\} - [[M^{zi}, A], A] = 0\,, \tag{A.1}$$

$$[M^{zi}, [M^{zj}, A]] + \{M^{iL}, M^{Lj}\} = -2\delta^{ij}B\,, \tag{A.2}$$

$$A = 2B + \frac{1}{2}M_{ij}^2 + \frac{d(d-2)}{4} - \langle Q_{AdS}\rangle\,, \tag{A.3}$$

$$[A, M^{ij}] = 0\,. \tag{A.4}$$

We note that Eq.(A.4) tells us that the operators A is invariant with respect to $so(d-3)$ rotations. Eq.(A.3) is because of the second order Casimir operator of $so(d-1,2)$ algebra is diagonal in irreps labelled by $D(E_0, \mathbf{h})$. Eqs.(A.1),(A.2) are consequences of commutators of the $so(d-1,2)$ algebra. Introducing new operator $\tilde{B}$ by relation

$$B = \tilde{B} + M^{zi}M^{zi}\,, \tag{A.5}$$

and plugging the representation for the operator A given in (A.3) in Eq. (A.2) we cast the above given system of equations into the following form

$$[[M^{zi}, \tilde{B}], \tilde{B}] + (M^3)^{[z|i]} + (\langle Q_{AdS}\rangle - \frac{1}{2}M^2 - \frac{d^2-5d+8}{2})M^{zi} = 0\,, \tag{A.6}$$

$$[M^{zi}, [M^{zj}, \tilde{B}]] + \delta^{ij}\tilde{B} = 0\,, \tag{A.7}$$

$$A = \frac{1}{2}M_{IJ}^2 + \frac{d(d-2)}{4} - \langle Q_{AdS}\rangle + 2\tilde{B} + M^{zi}M^{zi}\,, \tag{A.8}$$

$$[\tilde{B}, M^{ij}] = 0\,. \tag{A.9}$$

While deriving of these formulas we use commutation relations of $so(d-2)$ algebra spin operators $M^{IJ} = M^{ij}, M^{zi}$ given in (2.16) and exploit the relation

$$M^{IJ}M^{IJ} = M^{ij}M^{ij} + 2M^{zi}M^{zi} \,. \tag{A.10}$$

Our basic observation is that if we introduce the quantities

$$B^z \equiv \tilde{B}\,, \qquad\qquad B^i \equiv [\tilde{B}, M^{zi}]\,, \tag{A.11}$$

then the operator $B^I = B^i, B^z$ transform as a vector under rotation generated by $so(d-2)$ algebra. Indeed the second relation in (A.11) and Eqs.(A.7),(A.9) can be rewritten as

$$[M^{zi}, B^j] = \delta^{ij}B^z\,, \qquad [M^{zi}, B^z] = -B^i\,, \qquad [M^{ij}, B^z] = 0\,. \tag{A.12}$$

These commutation relations together with the ones following from $so(d-3)$ covariance

$$[M^{ij}, B^k] = \delta^{jk}B^i - \delta^{ik}B^j\,, \tag{A.13}$$

imply that the operator B^I is indeed transformed in vector representation of $so(d-2)$ algebra (cf. (2.21)). All that remains is to analyse Eq.(A.6). Making use of (A.11) we can rewrite (A.6) as

$$[B^z, B^i] + (M^3)^{[z|i]} + (\langle Q_{AdS}\rangle - \frac{1}{2}M^2 - \frac{d^2-5d+8}{2})M^{zi} = 0\,. \tag{A.14}$$

Now taking into account that a state obtainable by acting the spin operator M^{zj} on wave function $|\phi\rangle$ should also belong to $|\phi\rangle$ we conclude that constraint (A.14) should commute with M^{zj}. This gives a new constraint

$$[B^i, B^j] + (M^3)^{[i|j]} + (\langle Q_{AdS}\rangle - \frac{1}{2}M^2 - \frac{d^2-5d+8}{2})M^{ij} = 0\,. \tag{A.15}$$

Constraints (A.14),(A.15) are collected into the ones given in (2.20). Thus we proved that the old light cone formalism of Ref.[7] is equivalent to the one used in this paper. Remarkable features of new formalism are i) appearance of $so(d-2)$ vector B^I which was hidden in old formalism; ii) manifest $so(d-2)$ invariance of defining equations for B^I.

Appendix B Decompositions of tensor-spinor fields

Here we wish to describe an decomposition of totally symmetric tensor-spinor field $\Psi^{\hat{I}_1\ldots\hat{I}_s\alpha}$ transforming in irreps of the $so(d-1)$ algebra in terms of tensor-spinor fields $|\psi_{s'}\rangle$ which are irreps of the $so(d-2)$ algebra. Consider a generating function

$$|\Psi\rangle \equiv \Psi^{\hat{I}_1\ldots\hat{I}_s\alpha}\alpha^{\hat{I}_1}\ldots\alpha^{\hat{I}_s}|0\rangle\,, \qquad \alpha^{\hat{I}} = (\alpha^{1'}, \alpha^I)\,, \tag{B.1}$$

which satisfies the constraint

$$\alpha^{\hat{I}}\bar{\alpha}^{\hat{I}}|\Psi\rangle = s|\Psi\rangle\,, \qquad \bar{\alpha}^{\hat{I}}\bar{\alpha}^{\hat{I}}|\Psi\rangle = 0\,, \qquad \Gamma^{\hat{I}}\bar{\alpha}^{\hat{I}}|\Psi\rangle = 0\,, \tag{B.2}$$

where to define Γ-transversality constraint we use $\Gamma^{\hat{I}}$-symbols defined by relations

$$\Gamma^I = \gamma^I\,, \qquad \Gamma^{1'} = \left\{ \begin{array}{ll} \mathrm{i}^{(d-2)/2}\gamma^1\ldots\gamma^{d-2}\,, & \qquad d- \ \text{even;} \\ \mathrm{i} & \qquad d- \ \text{odd.} \end{array} \right. \tag{B.3}$$

We start with analysis of the second constraint in (B.2) which can be considered as the second order differential equation with respect to oscillator variable $\alpha^{1'}$

$$(\bar{\alpha}^{1'}\bar{\alpha}^{1'} + \omega^2)|\Psi(\alpha^{1'},\alpha^I)\rangle = 0\,, \qquad \omega^2 = \bar{\alpha}^I\bar{\alpha}^I\,. \tag{B.4}$$

Obvious solution to this equation is found to be

$$|\Psi(\alpha^{1'},\alpha^I)\rangle = \cos(\omega\alpha^{1'})|\Psi_s(\alpha^I)\rangle + \frac{\sin(\omega\alpha^{1'})}{\omega}|\Psi_{s-1}(\alpha^I)\rangle\,, \tag{B.5}$$

where $|\Psi_s\rangle$ and $|\Psi_{s-1}\rangle$ are rank s and $s-1$ traceful tensor-spinors, i.e. they are reducible representations of the $so(d-2)$ algebra. The solution (B.5) reflects well know fact that symmetric traceless rank s tensor of $so(d-1)$ algebra can be decomposed into symmetric rank s and $s-1$ traceful tensors of $so(d-2)$ algebra. The $|\Psi_s\rangle$,$|\Psi_{s-1}\rangle$ satisfy the constraints $(\alpha^I\bar{\alpha}^I - s)|\Psi_s\rangle = 0$, $(\alpha^I\bar{\alpha}^I - s+1)|\Psi_{s-1}\rangle = 0$. Now plugging (B.5) into the third constraint in (B.2) we find a relation

$$|\Psi_{s-1}\rangle = \bar{\alpha}^I\gamma^I\Gamma^{1'}|\Psi_s\rangle,. \tag{B.6}$$

Taking into account (B.6) and (B.5) we get a relation

$$|\Psi\rangle = \exp(\alpha^{1'}\bar{\alpha}^I\gamma^I\Gamma^{1'})|\Psi_s\rangle\,, \tag{B.7}$$

which tells us that traceless and Γ-transversal tensor-spinor field $|\Psi\rangle$ (see constraints (B.2)) is expressible in terms of one tracefull tensor-spinor $|\Psi_s\rangle$. This tracefull tensor-spinor field in turn can be decomposed into traceless tensor-spinors fields of $so(d-2)$ algebra $|\tilde{\psi}_{s'}\rangle$

$$|\Psi_s\rangle = \sum_{s'}\oplus|\tilde{\psi}_{s'}\rangle\,, \qquad\qquad s' = s, s-2, s-4,\ldots, s-2[s/2]\,. \tag{B.8}$$

The traceless tensor-spinor fields of $so(d-2)$ algebra $|\tilde{\psi}_{s'}\rangle$ satisfy by definition the constraints

$$\bar{\alpha}^I\bar{\alpha}^I|\tilde{\psi}_{s'}\rangle = 0\,, \qquad (\alpha\bar{\alpha} - s')|\tilde{\psi}_{s'}\rangle = 0\,. \tag{B.9}$$

Because the tensor-spinor $|\tilde{\psi}_{s'}\rangle$ does not satisfy γ-transversality constraint this field is a reducible representation of the $so(d-2)$ algebra. It can be decomposed into two irreducible representations of the $so(d-2)$ algebra by formulas

$$|\psi_{s'}\rangle = (1 - \frac{(\gamma\alpha)(\gamma\bar{\alpha})}{2s'+N-2})|\tilde{\psi}_{s'}\rangle\,, \qquad\qquad |\psi_{s'-1}\rangle = \gamma\bar{\alpha}|\tilde{\psi}_{s'}\rangle\,. \tag{B.10}$$

It is the set of traceless and γ-transversal tensor-spinor fields $|\psi_{s'}\rangle$ that we used to formulate light cone action for massive fermionic representations of $so(d-1,2)$ algebra.

References

[1] M. A. Vasiliev, Fortsch. Phys. **35**, 741 (1987)

[2] R. R. Metsaev, "IIB supergravity and various aspects of light-cone formalism in AdS space-time," arXiv:hep-th/0002008. Talk given at International Workshop on Supersymmetries and Quantum Symmetries, Moscow, Russia, 27-31 Jul 1999.

[3] J. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1998)] [arXiv:hep-th/9711200].

[4] A. A. Tseytlin, arXiv:hep-th/0311139.

[5] R. R. Metsaev, Phys.Lett.B**354**, 78(1995); Phys.Lett.B**419** ,49(1998) hep-th/9802097

[6] R. R. Metsaev, "Arbitrary spin massless bosonic fields in d-dimensional anti-de Sitter space," arXiv:hep-th/9810231. Talk given at International Seminar on Supersymmetries and Quantum Symmetries, Dubna, Russia, 22-26 Jul 1997.

[7] R. R. Metsaev, Nucl. Phys. B **563**, 295 (1999) [arXiv:hep-th/9906217].

[8] R. R. Metsaev, Nucl. Phys. Proc. Suppl. **102**, 100 (2001) [arXiv:hep-th/0103088].

[9] S. Deser and A. Waldron, Phys. Rev. Lett. **87**, 031601 (2001) [arXiv:hep-th/0102166]; Nucl. Phys. B **607**, 577 (2001) [arXiv:hep-th/0103198].

[10] Y. M. Zinoviev, arXiv:hep-th/0108192.

[11] M. Gunaydin, D. Minic and M. Zagermann, Nucl. Phys. B **534**, 96 (1998) [Erratum-ibid. B **538**, 531 (1998)] [arXiv:hep-th/9806042].

[12] E. Sezgin and P. Sundell, JHEP **0109**, 025 (2001) [arXiv:hep-th/0107186].

[13] R. R. Metsaev, Phys. Lett. B **531**, 152 (2002) [arXiv:hep-th/0201226].

[14] R. R. Metsaev and A. A. Tseytlin, Phys. Rev. D **63**, 046002 (2001); hep-th/0007036

[15] M.B. Green, J. H. Schwarz and L. Brink, Nucl. Phys. **B219**, 437 (1983).

[16] W. S. l'Yi, Phys. Lett. B **448**, 218 (1999) [arXiv:hep-th/9811097].

[17] A. Polishchuk, JHEP **9907**, 007 (1999) [arXiv:hep-th/9905048].

[18] A. Volovich, JHEP **9809**, 022 (1998) [arXiv:hep-th/9809009].

[19] A. S. Koshelev and O. A. Rytchkov, Phys. Lett. B **450**, 368 (1999) hep-th/9812238

[20] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

[21] M. Henningson and K. Sfetsos, Phys. Lett. B **431**, 63 (1998) [arXiv:hep-th/9803251].

[22] L. Brink, R. R. Metsaev and M. A. Vasiliev, Nucl. Phys. B **586**, 183 (2000) [arXiv:hep-th/0005136].

[23] K. B. Alkalaev, O. V. Shaynkman and M. A. Vasiliev, arXiv:hep-th/0311164.

[24] P. de Medeiros, arXiv:hep-th/0311254.

[25] V. K. Dobrev, arXiv:hep-th/0207116.

[26] M. Bianchi, J. F. Morales and H. Samtleben, JHEP **0307**, 062 (2003) hep-th/0305052; N. Beisert, M. Bianchi, J. F. Morales and H. Samtleben, hep-th/0310292.

[27] T. Biswas and W. Siegel, JHEP **0207**, 005 (2002) [arXiv:hep-th/0203115].

[28] X. Bekaert, I. L. Buchbinder, A. Pashnev and M. Tsulaia, arXiv:hep-th/0312252.